The effect of strong interactions on quarks' electromagnetic vector vertex

Qiaoli Yang

Department of Physics, Nanjing University, Nanjing 210093, P. R. China

Abstract

This paper studies the correction of electromagnetic vector vertex of quarks due to the strong interactions.

Key-words: Nonperturbative vector vertex; Global color symmetry model

Since quarks carry both electric charges and color charges, an interesting question will be how the strong interactions disturbs quarks' electromagnetic vector vertex. For electrons, the tree-level electromagnetic vector function is a simple γ_μ, thus the corresponding "Dirac" magnetic moment, which is usually expressed in term of the Lande g-factor, is: 2. However, higher order Feynman diagrams will differ the e-m vector vertex from a pure γ_μ, then g will slightly differ from 2. This is well known as the "anomalous magnetic moment" of electron. Composite particles often have a much huger anomalous magnetic moment such as proton. This indicates a great difference of e-m vertex function from simple γ_μ.

In ordinary state free quarks do not exist, however it may exist in strange matter under special environments such as in neutron stars. So, it will useful to consider the "anomalous magnetic moment" of quarks. This is essentially to calculate the e-m vertex function of quarks with both higher order e-m feynman diagrams and effects of the strong interactions. The higher order e-m feynman diagrams are always considered as small perturbation, so we can safely concentrate to latter contribution. In this paper I calculate the e-m vertex function of free quark with a relatively simple model "GCM" to describe the strong interactions, and employ Dyson-schwinger equation as a non-perturbation method for strong interactions calculation. (the Euclidean space field formulation is also employed for calculation convenience in future studies; The question of how one may proceed between Euclidean and Minkowski space is discussed by[1].)

The GCM generating functional of massless quarks in Euclidean space can be written as

$$Z[\bar{\eta},\eta,A_\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q\ e^{\left\{-\int d^4x\ \bar{q}(x)[\gamma\cdot\partial+\gamma_\nu A_\nu(x)]q(x) - \int d^4x d^4y\ \frac{1}{2}j^a_\mu(x)D_{\mu\nu}(x-y)j^a_\nu(y) + \int d^4x\ (\bar{\eta}q+\bar{q}\eta)\right\}} \tag{1}$$

here $j^a_\mu(x)$ denotes the color octet vector current $j^a_\mu(x) = \bar{q}(x)\gamma_\mu\frac{\lambda^a_C}{2}q(x)$ and $D_{\mu\nu}(x-y) = \delta_{\mu\nu}D(x-y)$ denotes the gluon two-point function.

The coordinate space vector vertex is:

$$\Gamma_\mu(x_1,x_2,y) = \frac{\delta^3\Gamma(A,\bar{q}^{cl},q^{cl})}{\delta A_\mu(y)\delta\bar{q}^{cl}(x_1)\delta q^{cl}(x_2)}\Big|_{A_\mu=0} \tag{2}$$

Where $\Gamma(A,\bar{q}^{cl},q^{cl})$ denotes the effective action. From the relation:

$$\frac{\delta^2\Gamma(A,\bar{q}^{cl},q^{cl})}{\delta\bar{q}^{cl}(x_1)\delta q^{cl}(x_2)} = g^{-1}[A_\mu]$$

Eq.(2) can be expressed as:

$$\Gamma_\mu(x_1,x_2,y) = \frac{\delta g^{-1}[A_\mu]}{\delta A_\mu(y)}\Big|_{A_\mu=0} \tag{3}$$

here $g^{-1}[A_\mu]$ denotes the inverse quark propagator under external field A_μ and can be expanded in powers of A_μ as follows:

$$
\begin{aligned}
g^{-1}[A_\mu] &= g^{-1}[A_\mu]\Big|_{A_\mu=0} + \int d^4y \frac{\delta g^{-1}[A_\mu]}{\delta A_\mu(y)}\Big|_{A_\mu=0} A_\mu(y) + ... \\
&= G^{-1}(x_1, x_2) + \int d^4y \Gamma_\mu(x_1, x_2, y) A_\mu(y) + ... \qquad (4)
\end{aligned}
$$

Which leads to the formal expansion

$$g[A_\mu] = G - \int d^4y G A_\mu \Gamma_\mu G + ... \qquad (5)$$

The dressed quark propagator has the decomposition in momentum space:

$$G^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2)$$

where the functions $A(p^2)$ and $B(p^2)$ are determined by the rainbow Dyson-Schwinger equation.

In momentum space, Eq.(4), Eq.(5) lead to:

$$G^{-1} \approx g^{-1}[A_\mu] - \Gamma_\mu A_\mu$$

$$G_0^{-1} \approx g_0^{-1}[A_\mu] - \Gamma_\mu^0 A_\mu$$

and

$$G \approx g[A_\mu] + G A_\mu \Gamma_\mu G$$

Insert them to the rainbow equation

$$G^{-1} = G_0^{-1} + \frac{4}{3} \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(p-k) \gamma_\nu G(k) \gamma_\mu$$

and up to the first order, one gets:

$$\Gamma_\mu(p, q) = \Gamma_\mu^0 - \frac{4}{3} \int \frac{d^4k}{(2\pi)^4} D(p-k) \gamma_\nu G(k + q/2) \Gamma_\mu(k, q) G(k - q/2) \gamma_\nu \qquad (6)$$

Γ_μ should be constrained by the vector Ward-Takahashi identity:

$$q_\mu \Gamma_\mu(p, q) = G^{-1}(P + \frac{q}{2}) - G^{-1}(p - \frac{q}{2}) \qquad (7)$$

Since Γ_μ transforms as a vector, and contains parameters p and q, it can be written as[3]:

$$\Gamma_\mu(p, q) = \Lambda_1(p, q) \gamma_\mu + \Lambda_2(p, q) p_\mu + \Lambda_3(p, q) q_\mu \qquad (8)$$

Insert Eq.(8) to Eq.(7):

$$q_\mu \Gamma_\mu = \Lambda_1 q_\mu \gamma_\mu + \Lambda_2 p \cdot q + \Lambda_3 q^2 \; = \; i\gamma \cdot (p + \frac{q}{2}) A\Big((p + \frac{q}{2})^2\Big) + B\Big((p + \frac{q}{2})^2\Big) - \Big\{ i\gamma \cdot (p - \frac{q}{2}) A\Big((p - \frac{q}{2})^2\Big) + B\Big((p - \frac{q}{2})^2\Big)\Big\} \tag{9}$$

Take traces of Eq.(9) of both sides:

$$\Lambda_2 p \cdot q + \Lambda_3 q^2 = B\Big((p + \frac{q}{2})^2\Big) - B\Big((p - \frac{q}{2})^2\Big) \tag{10}$$

Dot p_μ on both sides of Eq.(6) one gets:

$$\begin{aligned} & p_\mu \Gamma_\mu = \Lambda_1 p_\mu \gamma_\mu + \Lambda_2 p^2 + \Lambda_3 p \cdot q \\ & = p_\mu \gamma_\mu - \frac{4}{3} p_\mu \int \frac{d^4 k}{(2\pi)^4} D(p-k) \gamma_\nu G_1 \Gamma_\mu G_2 \gamma_\nu \end{aligned} \tag{11}$$

Dot γ_μ leads to:

$$\begin{aligned} & \gamma_\mu \Gamma_\mu = \Lambda_1 \gamma_\mu^2 + \Lambda_2 \gamma_\mu p_\mu + \Lambda_3 \gamma_\mu q_\mu \\ & = \gamma_\mu^2 - \frac{4}{3} \gamma_\mu \int \frac{d^4 k}{(2\pi)^4} D(p-k) \gamma_\nu G_1 \Gamma_\mu G_2 \gamma_\nu \end{aligned} \tag{12}$$

To express above equations clearer I first evaluate expression

$$\gamma_\nu G(k + \frac{q}{2}) \Gamma_\mu(k, q) G(k - \frac{q}{2}) \gamma_\nu$$

put expression of $G(p)$ in it and do some calculations:

$$\begin{aligned} & \gamma_\nu G(p_1) \Gamma_\mu(k, q) G(p_2) \gamma_\nu \\ & = \gamma_\nu \frac{1}{i\gamma \cdot p_1 A(p_1^2) + B(p_1^2)} [\Lambda_1 \gamma_\mu + \Lambda_2 k_\mu + \Lambda_3 q_\mu] \frac{1}{i\gamma \cdot p_2 A(p_2^2) + B(p_2^2)} \gamma_\nu \\ & = \frac{1}{[p_1^2 A^2(p_1^2) + B^2(p_1^2)][p_2^2 A^2(p_2^2) + B^2(p_2^2)]} \{I + II + III\} \end{aligned} \tag{13}$$

Here $p_1 = k + \frac{q}{2}, p_2 = k - \frac{q}{2}$ and:

$$\begin{aligned} I = \Lambda_1 \{ & 2A(p_1^2) A(p_2^2) (\gamma \cdot p_2) \gamma_\mu (\gamma \cdot p_1) - 4iA(p_1^2) B(p_2^2) p_{1\mu} \\ & - 4iA(p_2^2) B(p_1^2) p_{2\mu} - 2B(p_1^2) B(p_2^2) \gamma_\mu \} \end{aligned} \tag{14}$$

$$\begin{aligned} II = \Lambda_2 k_\mu \{ & -4A(p_1^2) A(p_2^2) p_1 \cdot p_2 + 2iA(p_1^2) B(p_2^2) (\gamma \cdot p_1) \\ & + 2iA(p_2^2) B(p_1^2) (\gamma \cdot p_2) + 4B(p_1^2) B(p_2^2)) \} \end{aligned} \tag{15}$$

$$
\begin{aligned}
III = \Lambda_3 q_\mu\{&-4A(p_1^2)A(p_2^2)p_1\cdot p_2 + 2iA(p_1^2)B(p_2^2)(\gamma\cdot p_1)\} \\
&+ 2iA(p_2^2)B(p_1^2)(\gamma\cdot p_2) + 4B(p_1^2)B(p_2^2))\}
\end{aligned} \tag{16}
$$

In order to have a solution of Γ_μ function, insert expression(13) to Eq.(11), Eq.(12) and take traces on both sides:

$$
\begin{aligned}
4p^2\Lambda_2 + 4p\cdot q\Lambda_3 &= -\frac{4}{3}p_\mu \int \frac{d^4k}{(2\pi)^4} D(p-k) tr\{Eq.(13)\} \\
&= -\frac{4}{3}\int \frac{d^4k}{(2\pi)^4} D(p-k)\frac{16}{[p_1^2A^2(p_1^2)+B^2(p_1^2)][p_2^2A^2(p_2^2)+B^2(p_2^2)]} \\
&\quad * \Big\{ \Big[-iA(p_1^2)B(p_2^2)p\cdot p_1 - iA(p_2^2)B(p_1^2)p\cdot p_2\Big]\Lambda_1 \\
&\quad + p\cdot k\Big[-A(p_1^2)A(p_2^2)p_1\cdot p_2 + B(p_1^2)B(p_2^2)\Big]\Lambda_2 \\
&\quad + p\cdot q\Big[-A(p_1^2)A(p_2^2)p_1\cdot p_2 + B(p_1^2)B(p_2^2)\Big]\Lambda_3\Big\}
\end{aligned} \tag{17}
$$

and

$$
\begin{aligned}
16\Lambda_1 &= 16 - \frac{4}{3}\int \frac{d^4k}{(2\pi)^4} D(p-k) tr\{\gamma_\mu Eq.(13)\} \\
&= 16 - \frac{4}{3}\int \frac{d^4k}{(2\pi)^4} D(p-k)\frac{1}{[p_1^2A^2(p_1^2)+B^2(p_1^2)][p_2^2A^2(p_2^2)+B^2(p_2^2)]} \\
&* \Big\{ -16[A(p_1^2)A(p_2^2)p_1\cdot p_2 + 2B(p_1^2)B(p_2^2)]\Lambda_1 \\
&+ [(8iA(p_1^2)B(p_2^2)k\cdot p_1 + 8iA(p_2^2)B(p_1^2)k\cdot p_2)]\Lambda_2 \\
&+ [(8iA(p_1^2)B(p_2^2)k\cdot p_1 + 8iA(p_2^2)B(p_1^2)k\cdot p_2)]\Lambda_3\Big\}
\end{aligned} \tag{18}
$$

These two equations above combined with Eq.(10) and the rainbow Dyson-Schwing equation of functions $A(p^2)$ and $B(p^2)$

$$
\begin{aligned}
[A(p^2)-1]p^2 &= \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} D(p-q)\frac{A(q^2)p\cdot q}{q^2A^2(q^2)+B^2(q^2)} \\
B(p^2) &= \frac{16}{3}\int \frac{d^4q}{(2\pi)^4} D(p-q)\frac{B(q^2)}{q^2A^2(q^2)+B^2(q^2)}
\end{aligned} \tag{19}
$$

will in principle fully restrict the nonperturbative vector vertex of quarks. By numerically study Eq.(10) Eq.(17) Eq.(18) and Eq.(19) with appropriate gluon two-point function, one can get the nonperturbative vector vertex and quark propagater in Euclidean space.

In order to have a qualitative understanding of this method, a particularly simple model gluon two-point function is used as follows:

$$
D\Big(p-k\Big) = g\delta^4(p-k) \tag{20}
$$

when $q \ll p$, we have $p + q/2 \approx p - q/2 \approx p$, Eq(10), Eq(17), Eq(18) have a simple solution of Λ_i with parameters $A(p^2)$ *and* $B(p^2)$:

$$
\begin{aligned}
\Lambda_1 &= \frac{12\pi^4[B^2 + 3\pi^4(A^2p^2 + B^2)^2/g - A^2p^2]}{g + 36\pi^8(A^2p^2 + B^2)^2/g - 15\pi^4A^2p^2 + 6\pi^4B^2} \\
\Lambda_2 &= \frac{24iAB\pi^4}{g + 36\pi^8(A^2p^2 + B^2)^2/g - 15\pi^4A^2p^2 + 6\pi^4B^2} \\
\Lambda_3 &= \frac{24iAB\pi^4 p\cdot q}{\Big[g + 36\pi^8(A^2p^2 + B^2)^2/g - 15\pi^4A^2p^2 + 6\pi^4B^2\Big]q^2}
\end{aligned}
\tag{21}
$$

Here A, B are solutions of the rainbow equation(19)

Insert the same two-point function of gluon to Eq.(19):

$$B(p^2) = 0, \quad A(p^2) = \frac{1}{2}\Big[1 - \Big(1 + \frac{2g}{3\pi^4p^2}\Big)^{1/2}\Big] \tag{22}$$

$$B(p^2) = 0, \quad A(p^2) = \frac{1}{2}\Big[1 + \Big(1 + \frac{2g}{3\pi^4p^2}\Big)^{1/2}\Big] \tag{23}$$

and:

$$B(p^2) = (\frac{g}{3\pi^4} - 4p^2)^{1/2},\ A(p^2) = 2 \ \ for \ \ p^2 \leq \frac{g}{12\pi^4} \tag{24}$$

Solution(24) generates a dynamical quark mass and breaks chiral symmetry spontaneously. Insert this solution to Eq.(21), I get the e-m vertex function of quarks:

$$\Gamma_\mu = \frac{8}{7}\gamma_\mu + \frac{16i}{7(\frac{g}{3\pi^4} - 4p^2)^{1/2}}p_\mu + \frac{16ip\cdot q}{7(\frac{g}{3\pi^4} - 4p^2)^{1/2}q^2}q_\mu \tag{25}$$

Acknowledgments

I would like to express my deep thanks to people who have ever helped me, especially to professor Hong-shi Zong and professor Fan Wang.

References

[1] Craig D. Roberts and Anthony G. Williams, arXiv: hep-ph/9403224 11 Aug 1997.

[2]Hong-shi Zong, Xiang-song Chen, Fan Wang, Chao-Hsi Chang and En-guang zhao, Phys. Rev. C66, 015201(2002).

[3] Michael E. Peskin, Daniel V. Schroeder, An Introduction to Quantum Field Theory, page 186.